Hand this part in before getting your calculator

- Give solutions, not just answers: provide adequate working or explanation, and circle your answer.
- The point total is 200.

1. (20 points) Find the limit, the infinite limit, or state that it does not exist. **Show your work or reasoning.**
   (a) \( \lim_{x \to 3} \frac{x - 3}{x^2 - 9} \)
   (b) \( \lim_{x \to 0} \frac{\sin(3x)}{4x} \)
   (c) \( \lim_{x \to -2} \frac{3 - \sqrt{x + 11}}{x + 2} \)
   (d) Given that \( \lim_{x \to -1} f(x) = 7, \lim_{x \to 2} g(x) = -4 \), find \( \lim_{x \to 1} [3f(x) - 5g(x)] \)
   (e) \( \lim_{x \to \infty} \frac{2 + \ln(x^2)}{7 + \ln(x^2)} \)

2. (10 points) Use the **definition** of derivative in terms of limits to find \( f'(2) \) where
   \[ f(x) = \frac{1}{x + 1} \]
   (You may check your answer by using the quotient rule, power rule, etc., but to receive credit for this question you must use the definition of derivative and evaluate the limit algebraically.)

3. (27 points) In this question you will differentiate and identify the graphs of the three functions
   \[ p(x) = -x^2 - x^2 - 5x \quad q(x) = xe^x \quad r(x) = -1 + \sqrt{x + 1}. \]
   (a) Write the first derivative of each function:
      \[ p'(x) = \]
      \[ q'(x) = \]
      \[ r'(x) = \]
   (b) Write the second derivative of each function:
      \[ p''(x) = \]
      \[ q''(x) = \]
      \[ r''(x) = \]
   (c) Evaluate your answers at \( x = 0 \):
      \[ p'(0) = \]
      \[ p''(0) = \]
      \[ q'(0) = \]
      \[ q''(0) = \]
      \[ r'(0) = \]
      \[ r''(0) = \]

4. (6 points) The temperature, \( T \), in degrees Fahrenheit, of a potato in a hot oven is given by \( T = f(t) \), where \( t \) is the time in minutes since the potato was put in the oven.
   (a) What are the units of \( f'(20) \)?
   (b) What is the practical meaning of the statement \( f'(20) = 2 \)?

5. (12 points)
   (a) Find \( \frac{dy}{dx} \) if \( y = \ln(3x^2 + 5) + \sin(e^{2x} + 2) \). (Do not simplify your answer.)
   (b) Find the derivative of the function \( f(x) = \frac{x}{x^2 + 4} \).
   (c) Evaluate \( \frac{d}{dx} \left[ \int_1^x \ln (1 + \sqrt{t}) \; dt \right] \).

6. (13 points) Find the equation of the tangent line to the curve
   \[ x^2 \sqrt{y - 2} = y^2 - 3x + y - 8 \]
   at the point \( (1, 3) \). (You may leave the equation in point-slope form.)

7. (20 points) For the function \( f(x) = 3x^4 - 8x^3 + 6x^2 - 3 \):
   (a) Find all of the critical points.
   (b) State clearly where the function is increasing or decreasing; give intervals or use a sign diagram.
   (c) Find all the maximum and minimum points, giving their \( x \) coordinates.
   (d) State clearly where the function is concave up and where it is concave down; give intervals or use a sign diagram.
   (e) Find all the \( x \) values at which \( f \) has a point of inflection.

   **Note:** It is not necessary to sketch the graph.

8. (10 points) Find the absolute maximum and absolute minimum values of \( f(x) = x^2 + \frac{16}{x} \) on the interval \([1, 8]\).
(9) (10 points) Sketch a possible graph of a function $f$ that has all the following properties:

a: $f$ is everywhere continuous;
b: $f(-1) = -2$, $f(0) = 0$
c: $f'$ does not exist at $x = -1$;
d: $f'(x) < 0$ for $x < -1$ or $x > 0$, $f'(x) > 0$ for $-1 < x < 0$;
e: $f''(x) < 0$ for $x < -1$ or $-1 < x < 1$, $f''(x) > 0$ for $x > 1$;
f: $\lim_{x \to -\infty} f(x) = 0$, $\lim_{x \to 0^+} f(x) = -2$.

(iv) $g$ has an inflection point there
(v) None of the above
(c) From this graph we can tell that $g'(3.5)$ is (circle one):
   - (i) positive
   - (ii) negative
   - (iii) zero
   - (iv) undefined

(11) (12 points) Evaluate each of the following definite integrals. You must show at least one intermediate step in your computation to get credit. You do not need to simplify your answers.

(a) $\int_1^2 (x^2 - 2) \, dx$
(b) $\int_0^1 \cos(x) \left( \sin^2(x) - 2 \sin(x) + 1 \right) \, dx$
(c) $\int_0^4 |x - 1| (x - 4) \, dx$

(12) (20 points) Find the most general indefinite integral. If you use a substitution, show your work. You do not need to simplify your answers.

(a) $\int (7 + 3 \cos(t)) \, dt$
(b) $\int \left( \sqrt{\frac{t}{3}} - \frac{1}{t} \right) \, dt$
(c) $\int \frac{2x^4 + 1}{x^2} \, dx$
(d) $\int \frac{2}{x^2 - 1} \, dx$
(e) $\int x \sqrt{4x - 5} \, dx$

(10) (10 points) The function $f(t)$ is shown in the graph below. Note that the two regions above the $t$-axis each have area 3 while the two regions below the axis each have area 1.5. Use this information to answer the following questions about the function $g(x)$ defined by the formula $g(x) = \int_0^x f(t) \, dt$.

(a) Evaluate $g(1)$ and $g(4)$.

(b) What happens on the graph of $g(x)$ at $x = 3$? (Circle one)
   - (i) $g$ has a "sharp corner" there
   - (ii) $g$ has a local min there
   - (iii) $g$ has a local max there
13. (10 points) A spherical balloon is being inflated, with air entering at a rate of 30 cubic inches per second. When the radius is 10 inches, how fast is the radius increasing?

14. (10 points) A Norman window has a semicircular upper section sitting above a rectangular part of the same width, as shown. You wish to construct such a window of total area 20 square feet and shortest perimeter. What dimensions should it have?

15. (10 points)

   (a) Approximate the value of the definite integral

\[ \int_0^6 \frac{1}{1 + t^2} \, dt \]

by a Riemann sum using right endpoints and \( n = 3 \) sub-intervals.

(b) Based on the fact that the function \( f(t) = \frac{1}{1 + t^2} \) is a decreasing function on the interval \([0, 6]\), how would you say your answer in part (a) compares to the actual value of the integral?

(c) If \( f(t) = \frac{1}{1 + t^2} \) is the rate (in gallons/minute) at which water is flowing into a tank at \( t \) minutes, what does the value of the integral in part (a) tell you?